INTERSTELLAR MATERIAL IN GLOBULAR CLUSTERS

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IN
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by

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ABSTRACT

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David Dupuy Heerwagen, Master of Science, 1963

Thesis directed by: Uco van Wijk
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The chances of survival of interstellar material in globular clusters is investigated. The observational evaluations of Roberts (1960) and Idlis and Nickol'skii (1959) are used. The effects of ultraviolet radiation due to hot, blue stars in the center of the cluster and the effects of the radiation field in general are discussed in Chapter III. The reaction due to collisions of globular cluster clouds with material in the galactic disk is evaluated both from momentum and energy influx considerations, and the probability of ejection of clouds from the cluster is estimated in Chapter IV. It is suggested that the most serious uncertainty is introduced by the incompleteness factor for hot O stars in globular clusters and further observations of these hot stars are desired.

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CHAPTER I

INTRODUCTION

Two recent investigations have stirred again the controversy concerning the possible presence of obscuring matter in globular clusters. First, Idlis and Nikol'skii (1959) reported sighting tiny globule-like objects in the central region of the globular cluster closest to the sun, Messier 4. Roberts (1960) investigated obscuring, cloud-like objects which were discovered in 12 out of 32 high altitude globular clusters.

The arguments presented by these investigators that the obscuring material is an integral part of the globular clusters seem convincing. They both showed that the probability of such vacant or obscured regions appearing as a result of statistical fluctuations in stellar distribution of the cluster is extremely remote, if not impossible. Roberts also demonstrated that for such obscuration to be caused by intervening halo clouds would require a space density of such clouds much higher than in the solar neighborhood - a result again extremely unlikely. It was also noted that clouds or obscurations do not appear against the background of the elliptical galaxies.

In this investigation then, we start with the assumption that these objects are indeed obscuring medium in the clusters, to be analyzed concerning both their reaction with the galactic disk and the possible environment within the cluster which affects their formation and behavior. From the investigation of Roberts, we sense a possibility that, since not all the clusters show obscuring matter, there is a selection effect operating to make conditions for formation of such objects and their retention and persistence within the clusters more favorable in certain cases than in others. This approach then suggests the possibility that there may be small but significant differences between individual

clusters. However, the globular clusters seem to be a remarkably homogeneous group of objects with respect to certain properties. Von Hoerner's (1957) study of globular clusters showed that their standard deviation in luminosity is less than 1 absolute magnitude, that their deviation of diameters is approximately 2% of the mean diameter, and that their internal structures are very similar. This homogeneity suggests that we adopt a standard cluster model as the framework within which to make our order of magnitude calculations concerning the interstellar material. Because of the great amount of observation and study devoted to the cluster Messier 3, we have generally adopted this object as our model.

Of the many basic assumptions that must be made in an investigation of this sort, perhaps the most difficult is that concerning the physical state and concentration of matter in the galactic disk, lying outside the discrete clouds and galactic spiral arms. Results from the study of optical, interstellar absorption lines, emission nebulae, absorption and obscuration of starlight, and from radio astronomy work in both the continuum and with the 21-cm line have given a fairly consistent picture of the galactic structure in the dense regions. However, a search of the literature reveals that confusion and controversy presently exist on the state and density of matter between the arms.

In his study of absorption lines in the optical region, Stromgren (1948) implies that the non-cloud region is permeated with a HI medium of approximately 0.1 H atoms cm^{-3}. Spitzer (1949) suggests at the first symposium on cosmical gas dynamics that the interstellar medium is in pressure equilibrium except for the relatively small volume of expanding H II regions near hot stars. His model would require the diffuse intercloud medium to have a kinetic temperature of 10,000 ^{o}K, signifying that it is primarily ionized or HII. A summary of his model shows:

	Relative Volume	T	$N(cm^{-3})$	P
Medium of HII	95%	10^4 K	10^{-1}	10^{-13} dynes/cm^2
Clouds of HI	5%	10^2 K	10^{+1}	10^{-13} " "
Expanding Regions of HII	0.5%	10^4 K	10^{-1}	10^{-11} " "

However, later Spitzer (1954) states that the "atoms in this tenuous intercloud medium could be mostly neutral and still at a relatively high temperature". Dufay (1955) says that regions of diffuse hydrogen of 10^{-2}cm^{-3} concentration could be HI regions of temperature 1000 $^{\circ}$K or less, because of the long times (10^9 years) required to reach equilibrium at this density. A gas at this density cools at a very slow rate. However, on page 125 he remarks that much of the intercloud medium could be HII. Finally, the question of whether the medium is predominantly HI or HII is discussed at length in the Third Symposium on Cosmical Gas Dynamics held in 1958. Spitzer (1958) concludes that "in any case, this intercloud medium is probably at a high temperature, so that its pressure is equal to that in the denser, cooler HI regions. Whether this rarified gas between clouds, and not near an early type star, is HI or HII is an open question".

In all probability, the intercloud hydrogen of the galactic disk is not uniform in concentration, temperature, or degree of ionization. However, an upper limit to its concentration can be set reasonably as 0.1 cm^{-3} or one order of magnitude lower than the concentration of 1 H atom cm^{-3} assigned to the spiral arms by radio 21-cm line and optical, absorption line studies. Concentrations as low as 0.01 cm^{-3} can also be considered reasonable.

In this investigation, we are concerned with the gross dynamics of the encounter process between a dense cloud or globule of the globular cluster and a typical cloud or the intercloud medium of the galactic disk. To simplify our calculations and to enable us to obtain both order of magnitude answers and limiting parameters, we have assumed the intercloud medium to be neutral. In fact, by using the

macroscopic approach of transfer of momentum and kinetic energy between large regions of gas and ignoring the individual particle or microscopic behavior, we reason that the question of a neutral or an ionized condition for the inter-cloud medium, interacting with the neutral clouds of the cluster, is not a sensitive input to the final estimates. Bates and Griffing (1953) have calculated collisional ionization cross sections for the neutral-neutral and neutral-ionized interaction of hydrogen. They find, that at the lower energies of approximately the range of our problem, the ionization cross section of the two reactions are approximately the same.

Little of theory or experiment, applying to astrophysical problems of these densities, has been done for the case of plasma interactions. Alfvén (1960) has considered the case of neutral-ionized collision. He analyzes the case where a non-ionized gas collides with a plasma where the kinetic energy per particle of relative motion between the particles of the two media exceeds the ionization energy per particle of the neutral gas. He concludes that the neutral gas will be ionized and trapped in the magnetic field and the density of the plasma will increase. However, implicit in his treatment, and also in the two papers by Kahn (1957, 1958) concerning collision of two ionized gases, is the assumption that the two gases have comparable densities. We will find that the case which ultimately interests us in this investigation is the collision of a dense, neutral gas of relatively small volume with a highly tenuous gas of much larger extent.

Kahn finds when two rarified masses of non-ionized gas collide at high enough velocity that both gases will be ionized after a relatively shallow penetration, and that the counter streaming electrons will be stopped in a very short distance. This results in a stationary, oscillating electron gas through which protons are streaming in opposite directions. This so-called collective instability of the electrons and not the individual collisions between particles

is responsible generally for stopping the counter-streaming gases.

However, Kahn does assume equal density of particles in each stream. For the case where a wide difference exists in the concentrations of the two counter-streams, a momentum exchange treatment is appropriate to determine the resulting motion of the arrested, now unified conglomerate.

So we will restrict our attention in this problem to the question of what happens to the cloud of a globular cluster with respect to the cluster's standard of rest when it impacts with the medium of the galactic disk. We will neglect considerations of individual particle reaction.

CHAPTER II

DISCUSSION OF OBSERVATIONAL DATA AND ASSUMPTIONS

Idlis and Nikol'skii (1959) detected small obscurations in the central regions of M4. These globules appear twice as numerous photographed in the blue as they do in the red, thus following a rough $1/\lambda$ absorption law and indicating the presence of dust particles characteristic of the plane of the Galaxy. They estimated the following average properties for the globules: radius = 0.02 pc, optical thickness (τ) = 3, mass of dust = 0.001 solar mass ($\odot$), total mass = 0.1$\odot$, concentration of hydrogen atoms = $10^5 cm^{-3}$, total number of clouds = 140. They report these globules as concentrated toward the center of the cluster with maximum concentration at about 0.6 pc distance from the center.

On the other hand, Roberts (1960) has reported the detection of larger, more massive objects in 12 out of 32 high latitude globular clusters examined. A typical cloud is that reported in M 13 as having a projected image of 1 by 3 pc and a total photographic absorption of 5 magnitudes. Roberts estimates the mass of dust in this particular cloud as 2.6 $\odot$. Assuming the estimated 1% ratio of dust to gas in the Galaxy plane to hold for the globular clusters, he finds the total mass to be 260 $\odot$. Actually, for simplicity of calculations, this is assumed to be the weight of the atomic hydrogen atoms. This would amount to a hydrogen atom concentration of the order of $10^3 cm^{-3}$. He estimates that a typical cluster which shows these clouds would have about 1000 $\odot$ tied up in such objects. Roberts does not discuss the distribution or location of his clouds, other than to show by photograph that they generally appear at the cluster edge where the apparent density or luminosity appear to drop sharply. However, from his discussion of the dimensions of a cloud shown in a photograph of M 13, one can deduce that the cloud lies a minimum distance of about 8 to 10 pc from the cluster center. The procedure used to determine the 5 magnitude absorption is not discussed by Roberts and is probably a very rough estimate.

It is clear that the foregoing two investigators are concerned each with an entirely different object, differing by about two orders of magnitude in both density and radius. Accordingly, any discussion of the possible origins, lifetimes, or evolutions of interstellar material in globular clusters should concern itself and attempt to account for both classes of objects. Throughout this paper, those objects reported by Idlis and Nikol'skii will be called globules while those of Roberts will be termed clouds.

In making our analysis, we will adopt a detailed model of a typical globular cluster. The cluster, Messier 3, has been observed and studied in great detail by Sandage (1954, 1957) and others. Accordingly, we use this observational data as a basis for our study. Also, we adopt Oort and van Herk's (1959) dynamical model (d) of M3 since it corresponds so closely to its observed structure.

Interstellar material could come from any or all of three sources: (1) residual material left over after original condensation of the cluster, (2) material that has been ejected from evolving cluster stars, (3) material that has been picked up from the halo and galactic disk by the accretion of clouds within the cluster. As we see later, sources (1) and (3) contribute a negligible amount compared to (2). We adopt the results of Sandage for the evolution of M3 which show that approximately 100,000 ⊙ have been converted into interstellar material over about 10^{10} years lifetime. We also assume the material to have the cosmic proportion of elements, as adopted for interstellar material in the galactic disk.

Orbits of Globular Clusters- Most of the rough orbit calculations of globular clusters are based on the radial velocity measurements made by Mayall (1946) or by Kinman (1959). In this investigation we will use the more recent data furnished by Kinman. He finds that the clusters follow highly eccentric orbits, the average eccentricity being 0.8. It is interesting to note that a graph in a recent paper by Eggen, Lynden-Bell, and Sandage (1962) also shows an average eccentricity of about 0.8 for high velocity stars of the subdwarf variety, which from ultraviolet

excess measurements, appear to correspond to the main sequence of globular clusters.

Matsunami and collaborators (1959) have computed galactocentric distances for 94 of the 118 known clusters, using Lohmann's heliocentric distances. We find the average galactocentric distance to be 8.5 kiloparsecs (kpc) for their 94 clusters. Since we know that for highly eccentric orbits, an object is most apt to be found near apogalacticon, we make a rough estimate that the average apogalacticon distance (Q) is 10 kpc. Using the formula $Q = a(1+e)$, we find the semi-major axis to be about 5.5 kpc. Using the formula for the latus rectum $p = a(1-e^2)$, we find $p = 2$ kpc.

Having found the parameters for the average orbit, we use them to obtain the average velocity (relative to the galactic frame of refernece) of a cluster as it traverses the galactic disk. For this calculation we assume that the average orbital axis makes an angle of 45 degrees with the galactic disk, and we use the polar equation for an ellipse, $r = \frac{p}{1+e \cos \alpha}$ (where α = anomaly), solving for r at the two points of intersection. Upon finding $r = 4.5$ kpc and 1.3 kpc, we select $r = 3$ kpc as a typical intersection point on the galactic disk.

We use the Schmidt model galaxy (1956) for the distribution of mass in the Galaxy. We find the following values from his graph for potential (in km^2/sec^2) in the $(\bar{\omega}, z)$ plane: at $Q = 10$ kpc, $\Phi_Q = 30{,}000\ km^2/sec^2$; at $r = 3$ kpc, $\Phi_r = 80{,}000\ km^2/sec^2$. Adopting the value $V_Q = 60$ km/sec at 10 kpc and using the formula $\frac{1}{2}V_Q^2 - \Phi_Q = \frac{1}{2}V_r^2 - \Phi_r$, we find $V\ = 330$ km/sec.

We have made the above approximations for a "mean" orbit for a globular cluster, in order to arrive at some typical value for the velocity of a cluster (relative to the galactic center) as it traverses the disk. In reality the orbits are not closed as we will see in Chapter V. However, for the high e orbits with which we are concerned, the foregoing simplifications will give an adequate

estimate. We intend to use this typical value in our analysis of the probable reaction between clouds of the cluster and the interstellar medium of the disk during transit. We must recognize that any such analysis is by necessity, order of magnitude accuracy. We must realize that for each individual transit, since the disk clouds rotate with the circular velocity of the disk, the relative velocity for collision can vary from about zero to approximately 600 km/sec. For purposes of our analysis, we will select 300 km/sec as the mean relative velocity. We use 2×10^8 years as the period for the mean orbit.

Stability of Clouds and Globules- We have adopted Roberts's cloud as being a spherical object of 260 solar masses and 2 pc diameter. It is interesting to consider the gravitational stability of such an object. For this purpose we use Jean's gravitational instability criterion (1928), and find a rough limit that a self gravitating object must satisfy in order to be gravitationally stable and not break up under the result of its own thermal motions. In applying any such criterion, we assume the cloud to be isolated in space. If there were a surrounding medium at a finite pressure, then the minimum temperature for stability would be larger. The most convenient form of this criterion, which can also be derived from the virial theorem, is: $N > N_{cr} = 2.5 \times 10^3 . T^3/M^2$ This signifies that a body of mass M (in solar masses) at temperature T must have a density of more than $N_{cr}\,cm^{-3}$ of hydrogen atoms to be stable. For the 260 ⊙ cloud, we find the density of hydrogen atoms to be about 2700 cm^{-3}. For this cloud to be gravitationally stable, its temperature would have to be below 40° to 50 $^\circ K$. Such a low temperature is certainly feasible in an HI region (neutral hydrogen) particularly when appreciable obscuring dust is present, and there is a strong possibility that any such object is stable. On the other hand, the 0.1 ⊙ globules fall short of the critical density. However, Idlis

has shown that these objects should have a lifetime of approximately 10^{11} years when considering their thermal diffusion, so the globules are probably long-lived objects in their present state.

CHAPTER III

ENVIRONMENT WITHIN THE GLOBULAR CLUSTER

If, in fact, we are convinced that the clouds and globules do exist in certain globular clusters, then the next matter of interest is an analysis of the environment within these clusters which encourages the formation and existence of such objects. Of principle interest is an estimate of the amount radiation from the stars beyond the Lyman limit (912 Å), which would ionize the hydrogen. Again, we use the observational data available for M3.

Star counts by Sandage for M3 (1954) have shown that the group of stars brighter than absolute visual magnitude +3.5 (1.25 ⊙) is slightly concentrated toward the center of the cluster. It is in this group of more massive stars above the main sequence that we expect to find those actively evolving toward the white dwarf stage by ejecting material. We might, therefore, expect to find this matter initially with the same distribution as these bright stars. Generally, this would mean a spherical distribution, somewhat concentrated toward the center. More specifically, we can give Roberts's estimate that a maximum of about 1000 solar masses of interstellar material is available between passages through the galactic disk. He makes this estimate for a recent epoch by first finding the number of cluster stars (ΔN) lying above the main sequence for M3, using the luminosity function of Sandage (1957). He also computes the time (Δt) necessary for the star to evolve through this region of the color-magnitude diagram. Then after finding Δm, the average mass loss suffered by a star during evolution, (using 1.2 ⊙ for initial mass and 0.6 ⊙ for final white dwarf mass) he estimates the mass loss of the stars of the cluster for a half period of its galactic orbit ($\frac{1}{2}T$) by the product: $\Delta N \cdot \left(\frac{\Delta m}{\Delta t}\right) \cdot \frac{1}{2}T$. Since more than half of these bright stars lie within 8 pc of the center of our M3 model, we assume as a first approximation that we have 700 ⊙ of diffuse material spread evenly within this radius.

This approximation allows us to make use of the concept of "Stromgren Spheres"

from Stromgren (1939) to show the extent of ionized hydrogen around stars of types O5 to A0, assuming that the surrounding material is distributed uniformly and of density 1 H atom cm^{-3}. These tabulated radii may then be modified for regions of different concentration by multiplying by $N^{-2/3}$.

To make this estimate for our cluster model, we use the computations of Minkowski and Osterbrock (1959) for M3. They found that about 15 hot blue stars, of effective spectral type O5 and of considerably fainter absolute magnitude ($M = +2.3$) than ordinary Population I stars of the same effective temperature, would be responsible for most of the ionization. The uncertainty of the data is illustrated by the fact that only one of these O5 stars has been sighted and the rest are inferred from an incompleteness factor computed by Sandage (1954). These 15 stars are distributed within 28 parsecs of the center of the cluster. Since we are concerned in our calculation with the volume within 8 pc of the center, we correct for the lower number of stars by noting that for the Oort-van Herk model (d), the volume within 8 parsecs contains about 0.6 of the mass of the stars brighter than $M = +3.5$ contained within 28 pc. So we correct the Minkowski-Osterbrock equation for the final total ionized volume, $V = \frac{1.9\times10^6}{N^2}(pc^3)$ where N number density, cm^{-3}, of H atoms, and use $V = \frac{1.1\times10^6}{N^2}(pc^3)$. Stromgren (1948) has given a correction to his tabulated radii of ionized spheres for the dilution effect for stars that lie outside the interstellar medium. Using this correction, we find we can neglect the contribution of those blue stars outside of 8 pc for our order of magnitude estimate.

Now, using a total average mass of hydrogen atoms in diffuse matter within 8 pc as 700 ⊙, we find the average concentration of H atoms is $N = 15\ cm^{-3}$. At this density, we find that the whole volume within 8 pc would be ionized. This result is for a uniformly distributed medium, and does not take into account that any material would be concentrated toward the center as indicated by the potential field in Figure 1. Also there could and probably would be turbulence and density

fluctuations. Nonetheless, this particular environment does not appear conducive to the formation of HI regions, particularly of the size of 260 ☉. Ionized hydrogen has a kinetic temperature of approximately 10,000 °K, which gives a random thermal velocity of about 16 km/sec that would cause it to escape from the cluster quickly.

In order to form a general impression of the effect of more interstellar material within a globular cluster, we will hypothesize that 3,000 ☉ of material is available, which would be the case if the material remains undisturbed in the cluster for a time interval equal to three mean orbital periods. About 2,000 ☉ of this would be within 8 pc. We will again assume it to be spread uniformly. Using the relation $V = \frac{1.1x10^6}{N^2}(pc^3)$ with N now equal 42 cm^{-3}, we find that less than one-third of the volume within 8 pc, or 630 pc^{-3}, is now ionized. We could visualize, in this particular case, regions of ionized material or emission nebulae surrounding each hot, blue star, separated by larger volumes of HI gas. For the case between the two extremes just discussed, we might picture instead a region of tenuous, ionized gas escaping around floating, smaller volumes of HI.

The 2,000 ☉ of interstellar hydrogen would be just ionized, if distributed uniformly within a volume of 11 pc radius. It is interesting for this case to estimate the time interval for ionizing the entire 2,000 ☉, assuming that all quanta beyond the Lyman limit (912 Å) are fully absorbed by the hydrogen atoms. To make this approximation, we must first calculate the radius of the sub-luminous O-5 type star, which is the principal ionizing agent. We use a formula from Russell, Dugan, and Stewart (1938) $M_v = \frac{29,500}{T} - 5 \log R - 0.08$. From Stromgren, we adopt an effective temperature for the O5 star of 79,000 °K. Solving for R, we get 0.4 solar radii.

The energy density at the surface of a star of effective temperature, T, is given by the Planck law as;

$$u_\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{(e^{h\nu/kT} - 1)}$$

To find the energy density in the frequency range beyond the Lyman limit (ν_0), we integrate the above expression:

$$u = \frac{8\pi h}{c^3} \int_{\nu_0}^{\infty} \left(e^{h\nu/kT} - 1\right)^{-1} \nu^3 \, d\nu$$

At these high frequencies, we make use of the Wien approximation: $\left(\frac{h\nu_0}{kT} = 2\right)$

$$u = \frac{8\pi h}{c^3} \int_{\nu_0}^{\infty} e^{-h\nu/kT} \nu^3 \, d\nu$$

Integrating, we get:

$$u = \frac{8\pi}{c^3} \left(kT\nu_0^3 e^{-h\nu_0/kT}\right) \left[1 + 3\left(\frac{kT}{h\nu_0}\right) + 6\left(\frac{kT}{h\nu_0}\right)^2 + 6\left(\frac{kT}{h\nu_0}\right)^3\right]$$

Now, to find the intensity at the star's surface, we use the relation:

$$I = \frac{uc}{4} \left(\mathrm{erg/cm^2 sec}\right)$$

$$I = \frac{2\pi}{c^2} \left(kT\nu_0^3 e^{-A}\right)\left(1 + 3/A + 6/A^2 + 6/A^3\right)$$

where $A = h\nu_0/kT$

$$L\left(\mathrm{ergs/sec}\right) = 4\pi R^2 I \qquad L = 1.7 \times 10^{37} \ \mathrm{ergs/sec}$$

This is the luminosity beyond the Lyman limit for one 05 star of $M_v = +2.3$. We can check these figures by giving an approximate B.C. (bolometric correction) for an 05 star. From Allen (1955), BC = -42.5 + 10 log T + 29,000/T, and for T = 79,000 °K, BC = 6.9. Using the value, $M_{bol} = 4.62$ for the sun and $L_\odot = 3.86 \times 10^{33}$ ergs/sec., we find $L_{05} \cong 1.9 \times 10^{37}$ ergs/sec. This figure indicates that our calculated luminosity beyond the Lyman limit is of the correct order of magnitude.

Within 11 parsecs, we have approximately 11 (05) stars, so that the total energy output absorbed by 2000 ⊙ of hydrogen is 1.9×10^{38} ergs/sec. Each atom of hydrogen absorbs 13.54 ev in order to ionize. In 2000 ⊙, there are approx-

imately 2.4×10^{60} hydrogen atoms. This means we require:

$$2.4 \times 10^{60} \times 13.54 \text{ ev} \times 1.6 \times 10^{-12} \text{ ergs/ev} = 5.2 \times 10^{49} \text{ ergs}$$

to ionize all the material. At the energy output rate of 1.9×10^{38} ergs/sec, the ionization lifetime is:

$$\frac{5.2 \times 10^{49} \text{ ergs}}{1.9 \times 10^{38} \text{ ergs/sec} \times 3 \times 10^{7} \text{ sec/year}} \simeq 10^{4} \text{ years}$$

Such a short ionization time (this is, of course, a minimum limit because of neglect of radiation losses) would suggest that the interstellar material is almost immediately ionized as it appears and quickly lost from the cluster. However, all of our estimates have been based on the use of Stromgren spheres for a group of stars with the interstellar material uniformly distributed. In reality, we would not expect this to be the case. Space densities derived for M3 from Sandage's counts for stars brighter than $M_v = +3.5$ show that these brighter stars have a distribution proportional to R^{-3}. King (1962) has recently given an empirical law for projected densities of the inner regions of globular clusters substantiating this type of distribution. Assuming that the initial space density of the material follows the same law as the brighter stars, we conclude that the initial distribution of matter would be highly concentrated toward the center. If we also assume that the few stars responsible for most of the ionization follow the same R^{-3} law, then we can see that the central regions provide the likeliest environment for breeding HI clouds. The volume of ionization varies directly as the total luminosity of the ionizing stars and inversely as N^2. Since the total luminosity varies directly as the number of ionizing stars, we finally can write that the percentage of ionized volume varies as $\rho_{**}/N^2 = \frac{1}{\rho_{**}}$. In other words, the outer regions tend to ionize more easily. The foregoing arguments do not attempt to provide any mechanism to account for the actual formation of HI clouds, but merely offer a general analysis as to the most favorable regions for such formation.

An investigation by Spitzer (1942) shows that for a spherical self-gravitating system the interstellar material retained by a potential field would tend to concentrate toward the center because of the field and the effects of inelastic collisions. This effect would accentuate the initial concentration. The fact that Roberts's clouds are found some distance from the center seems to pose an anomaly. This point will be elaborated further in Chapter IV.

It is evident from the foregoing discussion that one of the biggest detriments to the formation of HI regions is the presence of hot, blue O stars with their high proportion of radiation beyond the Lyman limit. This would suggest that a correlation exists between the presence and distribution of O stars and the formation of HI regions. Any such correlation would not be straightforward, however, because once dense HI clouds would form, then the subsequent appearance of these sub luminous O stars in the process of evolution would not necessarily ionize dense regions. Nonetheless, it would seem logical that some correlation between the age of stage of evolution of a cluster and a favorable environment for the production of HI regions might exist. This point will be discussed further in Chapter V.

We next look briefly at the kinetic equilibrium temperatures we might expect to find in the HI clouds of the cluster. A series of papers by Spitzer and Savedoff (1948, 1949, 1950, 1955) have concluded that the equilibrium temperature of the HI clouds in the galactic plane is nearly independent of its density in the range from about 1 to 10^6 H atoms cm^{-3}. This conclusion assumes that the gas has sufficient time to reach equilibrium with its associated radiation field.

Before analyzing the HI regions in a globular cluster, we first might discuss some of the more important energy gain and loss mechanisms that are involved. These were discussed by Eddington (1930) and later, in more detail by Spitzer and Savedoff. Throughout the discussion, we should remember that

we are dealing with a medium not in thermodynamic equilibrium, and in a dilute radiation field. However, the frequent elastic encounters between the particles do tend to establish an equipartition of the kinetic energy of translation, and the resulting approach to a Maxwellian velocity distribution can be defined by an effective kinetic temperature.

We can assume that there is virtually no radiation in HI regions beyond the Lyman limit, 912 Å. The primary heating mechanism is the superelastic collisions of electrons with those atoms having ionization potentials below 13.54 volts and which are relatively abundant, i.e. C, Mg, S, Fe, Al. Heating by cosmic rays, at least in the solar neighborhood, is probably negligible for HI regions of 1 atom cm^{-3} or denser. The energy loss mechanisms are (1) inelastic collisions of electrons with ions (particularly CII and SiII), (2) inelastic collisions between H atoms and grains (3) inelastic collisions between electrons and H2 molecules, and H atoms and H2 molecules.

While Spitzer and Savedoff do not consider time-dependent sources, Kahn (1955) estimates that collisions between separate clouds could be even more effective than photoionization of the atoms as a heating mechanism. By their analysis, Spitzer and Savedoff arrive at an equilibrium temperature between 30 and 100 ^{o}K for HI clouds of density 10 cm^{-3} or more. At low densities between clouds where radiative cooling processes are ineffective, temperatures upward of 500 ^{o}K are postulated. Kahn shows that cloud temperatures can be expected to drop very rapidly down to 500 ^{o}K between collisions, and more slowly below 500 ^{o}K. These analyses are observationally substantiated by the 21-cm measurements which yield a harmonic mean temperature of about 100 to 125 ^{o}K for the HI clouds of the galactic plane.

In attempting to apply any such analysis to HI clouds in globular clusters, we are at once struck with the great uncertainties involved. Certainly, we can generally assume that the chemical composition of the medium in the clusters has

a lower proportion of the heavy elements than does gas in the galactic plane. Spectra of the globular clusters are interpreted to indicate metal to hydrogen ratios from 1/10 to 1/100 that of the sun. A lower percentage composition of the elements with low ionization potential would reduce the contribution of one heating mechanism for HI regions and perhaps lead to lower equilibrium temperatures.

We can also expect the radiation density inside a globular cluster to differ from that of the solar neighborhood. We can estimate this value quite readily. We use the following notation in calculating the approximate radiation density inside Messier 3:

u = radiation density (ergs/cm^3).

r = distance from center of cluster (cm).

c = velocity of light (3×10^{10} cm/sec).

$\bar{\ell}$ = average luminosity (ergs/sec) of those cluster stars brighter than $M_v = +3.5$.

L = total luminosity of cluster (ergs/sec)

$\mathcal{N}(r)$ = density of stars in cluster brighter than $M_v = +3.5$ (stars/parsec^3) at a distance r.

From the Oort-van Herk data for model (d) of M3, we use the value 3.3 for $L/\mathfrak{M}$ and 190,000 solar mass for the total mass ($\mathfrak{M}$) of the cluster. Using the value 3.86×10^{33} ergs/sec, for the total luminosity of the sun and neglecting the probably small bolometric correction for the cluster, we find:

$$L = (3.86 \times 10^{33} \text{ ergs/sec}) \cdot (190{,}000) \cdot (3.3) = 2.43 \times 10^{39} \text{ ergs/sec.}$$

For the purpose of determining the radiation density of the cluster, we consider that most of the light comes from those stars brighter than $M_v = +3.5$, so that the density distribution of these stars can be equated to the distribution of light sources. For a single star: $u = \dfrac{\bar{\ell}}{4\pi r^2 c}$ at any point a distance r from the star.

Now, to account for the contribution of radiation of all the other stars to this single point, we integrate over all the cluster, or in this case out to

100 pc, the visible limit.

So, at a single point in the cluster, we have:

$$(1)\quad u = \int_0^{100} \frac{\bar{\ell}}{4\pi r^2 c} (4\pi r^2 \nu)\, dr = \int_0^{100} \frac{\bar{\ell} \nu\, dr}{c}$$

For the total luminosity of the cluster, we have the following expression:

$$(2)\quad L = \int_0^{100} \bar{\ell}\, 4\pi r^2 \nu\, dr$$

Dividing (1) by (2), we get

$$u/L = \frac{\int_0^{100} \nu\, dr}{4\pi c \int_0^{100} r^2 \nu\, dr}$$

Our final expression for radiation density is then:

$$(3)\quad u = L/4\pi c \; \frac{\int_0^{100} \nu\, dr}{\int_0^{100} r^2 \nu\, dr}$$

Figures (2) and (3) from Oort-van Herk depict the space densities (ν) of the group of stars brighter than $M_v = +3.5$ versus cluster radius (r). These graphs are based on star counts for Messier 3 by Sandage and von Zeipel. In order to find u, we evaluate equation by carrying out the integrations. We split up the interval from 0 to 100 into 3 segments: (1) integrating numerically by Simpson's Rule from 0 to 4 psc, (2) represent analytically as $\nu = \frac{k_1}{r^3}$ from 4 to 7 parsecs, and (3) as $\nu = \frac{k_2}{r^{3.6}}$ from 7 to 100 parsecs.

Upon evaluating the two integrals, and using $L = 2.43 \times 10^{39}$ ergs/sec, we find that $u = 2.9 \times 10^{-10}$ ergs/cm^3.

This value can be compared with the density of dilute black body radiation as calculated for the solar neighborhood: 5.24×10^{-13} ergs/cm^3 (Dunham, 1939) or 7.67×10^{-13} ergs/cm^3 (Eddington, 1930), for which the black body temperature of a particle in space is 3.18 $^{\circ}$K.

Using the formula $u = akT^4$, where $a = 7.57 \times 10^{-15}$ ergs/cm^{3}deg^4 and k is the absorption coefficient, we can solve for the effective temperature of the dust in the cloud if we can find an expression for k.

We know from Roberts that the 260 ⊙ cloud absorbs approximately 5 magnitudes in the visible light ($\lambda \simeq 5 \times 10^{-5}$ cm). From Wien's displacement law, $\lambda_{max} = \frac{.29}{T}$

We can solve for λ_{max} for the effective temperature.

We also know that k varies as $1/\lambda$, so $k = \frac{5 \text{ mag. } \times 5 \times 10^{-5}}{\lambda_{max}}$.

We therefore have $k = \frac{25 \times 10^{-5} \times T}{.29}$, giving us $u = a \frac{25 \times 10^{-5}}{.29} T^5$. Substituting our value of $u = 2.9 \times 10^{-10}$ ergs/cm^3 and solving for T, we find $T \simeq 34$ °K.

The observational evidence supplied by Roberts (1959) by his 21-cm measurements of M13 implies an equilibrium temperature of 100 °K or below. Hence, we are led to conclude that the temperatures of dense HI clouds in globular clusters are at least as cool as HI clouds in the galactic plane.

CHAPTER IV

PASSAGE OF GLOBULAR CLUSTERS THROUGH THE GALACTIC DISK

We now turn to an investigation of the interaction between gas clouds and globules in the cluster with the medium lying in the galactic plane. Previous investigators have simply stated that any gas present in the cluster will be ejected by each passage through the disk.

The galactic disk consists of a thin layer of gas and dust distributed in a non-uniform fashion. There are individual clouds and spiral arms separated by regions of low density. Densities for each feature are: typical individual clouds - 10 atoms cm^{-3}, spiral arms - 1 atom cm^{-3}, space between clouds - 0.1 to 0.01 atoms cm^{-3}. As previously argued, we assume most of the hydrogen to be neutral for purposes of the collision process. The ionized gas in the disk is concentrated around the individual O and B stars.

Since we are interested in the processes that tend to act on the neutral hydrogen clouds and globules in the clusters, we can see that the collision to be studied is the HI-HI impact.

As long as the mean free path for particle collision is small compared to the dimensions of the cloud or globule, we can treat the gross dynamics of the encounter by evaluating the momentum transfer to determine if the clouds and globules of the cluster are ejected from inertial force considerations. For collisions of neutral H atoms, we can use the geometrical radius ($\sim 10^{-8}$cm) to compute the collision cross-section, giving $\sigma = 3\times10^{-16}\,cm^2$ (Parker, 1958). For the globules, $N_H \simeq 10^5\,cm^{-3}$; for the clouds, $N_H \simeq 10^3\,cm^{-3}$. Using the relation, λ(mean free path)$= 1/N\sigma$, we find λ glob$= 10^{-8}$pc and λ cloud$\simeq 10^{-6}$pc; with both values considerably smaller than the dimensions of the object involved.

Kahn (1955) has considered the statistics of size and distribution of clouds in the galactic plane by using the observational evidence that: (1) a line of sight of 1 kiloparsec intersects an average of 10 clouds, and (2) clouds occupy

about 5% of the volume of the galactic disk. He concludes that the average cloud is 3.75 parsecs radius, with a density of 20 H atoms cm^{-3} and that the number density of clouds in space is $2.45 \times 10^{-4} pc^{-3}$. This result, of course, is characteristic of the solar neighborhood. We readily see that the mean free path for collision is about 100 parsecs.

From 21 cm. observations, Rougoor and Oort (1960) have described the general galactic disk between 5 and 10 kpc from the galactic center as being 220 pc thick with a mean density of 1 H atom cm^{-3}. The disk inside 3 kpc seems to be much thinner, about 120 pc, and the mean density is given as 0.4 H atoms cm^{-3}. They comment that the region between 3 kpc and the central disk at 600 pc appears to be virtually empty.

From our discussion in Chapters II and III, we note the following information concerning the clouds and globules: (1) In general, we would expect to find HI regions toward the center of the cluster in an isolated cluster. (2) From the incomplete information now available, clouds seem to appear at the edge of the clusters (minimum of 8 to 10 pc) and globules toward the center of a cluster (in the one example, M4).

Let us assume that a globular cluster contains a 260 $\odot$ cloud of Roberts's description at the center of the cluster. Let us then analyze the possible effects of a collision with the galactic disk. In this analysis we will use two different treatments for the two different types of impact involved. If we assume that the cluster traverses a mean distance through the disk of 300 parsecs, then at 300 km/sec (where 1 km/sec $\simeq$ 1 parsec/10^6 years) we can conclude the time of transit is about 10^6 years. However, a cluster cloud would transit one of Kahn's typical, dense clouds in the galactic plane (diameter $\simeq$ 7.5 pc) in about 10^4 years. If we compare these two characteristic times with the approximate orbital period of an object in the potential field of the cluster, 10^7 years, we see the need for different treatments.

Momentum Transfer

For the case of impact with a discrete cloud in the disk, we consider an impulse to be acting on the cluster cloud. We will also assume that the collision is completely inelastic, the colliding portion of the disk cloud remaining with the cluster cloud.

For the case of impact with the intercloud medium (0.1 to 0.01 H atoms cm^{-3}) of the disk, we compute the momentum transfer per unit time of the colliding medium, which is the force acting on the cloud or globule of the cluster.

We will see that one effect of the collision with a disk cloud can be to knock a cluster cloud out of the cluster center and into a rectilinear orbit. On the other hand, the force exerted by the medium on a cluster cloud effectively shifts the whole orbit in the direction away from the direction of motion.

The simplest manner in which to visualize this problem is to analyze it with respect to the potential well of the cluster and to consider the motion as a one dimensional oscillator. We use the following notation:

V = velocity of globular cluster relative to the Galaxy (300 km/sec).

R = distance of cloud or globule from center of cluster.

r = radius of cloud or globule of cluster.

L = diameter of disk cloud (7.5 pc)

n = number density of H atoms in galactic disk (cm^{-3}).

m_H = mass of hydrogen atom (1.67×10^{-24} gm).

$\mathcal{M}$ = mass of cloud in cluster (260 $\odot$).

$\Phi(R)$ = potential as a function of R in the cluster $(km/sec)^2$.

$\Delta\Phi$ = potential gradient.

N = number density of H atoms in globular cluster cloud.

F = Momentum per unit time or force applied to cloud in cluster resulting from transit of intercloud medium of disk.

E = Total energy per unit mass of cloud in globular cluster. $(km/sec)^2$. (with respect to cluster frame of reference).

Figure (1) depicts our detailed example of the globular cluster with a 260 $\odot$ cloud at the center of the cluster on its first impact with the disk.

First, we derive a general formula for computing the force per unit mass resulting from momentum transfer per unit time from the intercloud medium to the cluster cloud on transit of the disk. We have that: $F = \pi r^2 . V^2 . n . m_H$ (dynes)

$$\mathcal{M} = \frac{4\pi r^3}{3} . N . m_H \text{ (grams)}$$

(1) $F/\mathcal{M} = \frac{3\ V^2 . n}{4\ r . N}$ (cm/sec^2)

For our particular case, where r = 1 pc and $N = 2.7.10^3 cm^{-3}$, V = 300 km/sec and $n = 0.1\ cm^{-3}$, equation (1) gives us: $F/\mathcal{M} = 8.2x10^{-9} cm/sec^2$ or $2.5\left(\frac{km}{sec}\right)^2 pc^{-1}$.

We can also give for our example, the expression for the force as:

(2) $F(\text{dynes}) = 4.25x10^{28} . n(cm^{-3})$

or $F = 4.25x10^{27}$ dynes (for n = 0.1).

We have the following relationships between total energy per unit mass (E), work done on the cluster cloud per unit mass $\frac{F.(R_2-R_1)}{\mathcal{M}}$ potential energy per unit mass (Φ) and kinetic energy per unit mass ($\frac{1}{2}v^2$), where v = velocity of cloud in its orbit with respect to the globular cluster:

(3) $E_2 = E_1 + \frac{F}{\mathcal{M}}(R_2-R_1)$ where E_1 is the total energy in an orbit at time t, and E_2 is the total energy at some time t_2 later. As shown in figures (1) and (2), $\dot{R}$ values in the direction away from the direction of impact are positive. We also have the following identity relating our energies:

$E \equiv \Phi + \frac{1}{2}v^2$ (where v is the velocity of the cloud in its orbit relative to the frame of reference of the cluster)

We can see that the value of $F/\mathcal{M} \propto \frac{V^2 n}{rN}$ determines to what extent an orbit is displaced to the rear because of the steady force exerted by the intercloud medium during about 10^6 years. In figure (2) we have depicted the effect of force per unit mass exerted by both a 0.1 cm^{-3} medium ($F/\mathcal{M} = 2.5(km/sec)^2 pc^{-1}$) and a 0.01 cm^{-3} medium ($F/\mathcal{M} = 0.25$). We depict the extreme case for both these

media in order for a cluster cloud (of Roberts's description) to be retained in the cluster for an indefinite period of time. For the 0.1 cm^{-3} medium, the limiting orbit is from -4 pc to +14 pc; for n = 0.01 cm^{-3}, it is from -20 pc to +50 pc.

We should note, however, that for our particular case where the time of transit of the disk is 10^6 years and the orbital period in the cluster is $\sim 10^7$ years, that the chance of a cloud escaping due to the reaction of the medium alone is extremely remote. Once the cluster emerges from its transit of the disk, then the $F/\mathcal{M}$ slope is practically zero and, as long as the cloud has a total energy less than 138 $(km/sec)^2$, it will not escape.

However, the reaction caused by collisions with discrete clouds in the galactic disk of the order of n = 10 cm^{-3} is extremely significant for our discussion. To calculate the energy change per unit mass (ΔE), we use the principle of conservation of momentum. For the general case we have for the mass of material intercepted by the cluster cloud:

$$\mathcal{M}_i = L.\pi.r^2.n.m_H$$

The mass ($\mathcal{M}$) of the cluster cloud is:

$$\mathcal{M} = 4/3\pi r^3.N.m_H$$

V is the relative velocity of impact (300 km/sec).

Δv is the velocity change imparted to the cluster cloud relative to the cluster frame of reference.

We then have: $\mathcal{M}_i.V = (\mathcal{M} + \mathcal{M}_i).\Delta v$

This gives the expression $\Delta v = \frac{V \cdot \mathcal{M}_i}{(\mathcal{M} + \mathcal{M}_i)}$

$$(4) \qquad \Delta v = \frac{V}{1 + \frac{4rN}{3Ln}}$$

Now we derive a general expression for the energy transfer per unit mass per collision. We have:

$E_i = \Phi + \frac{1}{2}v_i^2$ (where v_i is the orbital velocity of the cloud at time t_i)

$E_f = \Phi + \frac{1}{2}(v_i + \Delta v)^2$ (E_f = energy after impulse)

This gives us the general expression for energy transfer:

$$(5) \quad \Delta E = E_f - E_i = v_i \Delta v + \frac{(\Delta v)^2}{2}$$

Substituting the appropriate values into equation (4) for the 260 ⊙ cluster cloud colliding with the n = 10 cm^{-3} disk cloud, we find $\Delta v \simeq 6$ km/sec. From equation (5) then, we have $\Delta E = 6\ (v_i + 3)$.

Now, with the use of figure (1), let us follow through in some detail the reactions of a 260 ⊙ cloud, initially at rest in the center of a globular cluster, as it passes through a typical area of the galactic disk. We adopt 0.1 H atoms cm^{-3} as the concentration for the intercloud medium. We assume that the average cluster passes through 300 pc of the disk. The mean free path for collision with one of the disk clouds (L = 7.5 pc, n = 10 cm^{-3}) is 100 pc.

Upon gradual entry of the cluster into the 0.1 cm^{-3} intercloud medium, we expect the cloud to be shifted about 0.2 pc to the rear of the cluster center, or to R = +0.2 pc. The cluster cloud is likely to be near this point on the diagram upon its first collision with a disk cloud. Using our previously calculated value of $\Delta v = 6$ km/sec, we find from equation (5), where $v_i = 0$, $\Delta E = 18\left(\frac{km}{sec}\right)^2$. We indicate this change in energy on figure (1) by a dotted vertical line to point (R_1, E_1) or (+0.2, 19). At this point we assume that about one third of the transit period ($\sim 3\times10^5$ years) has elapsed. Through this point we draw a line with the proper slope to represent $F/m \simeq 2.5\ \left(\frac{km}{sec}\right)^2 pc^{-1}$, as previously calculated for the force per unit mass exerted by the 0.1 cm^{-3} medium on the 260 ⊙ cloud. This line represents the locus of (R,E) points for the rectilinear orbit which the cluster cloud now follows.

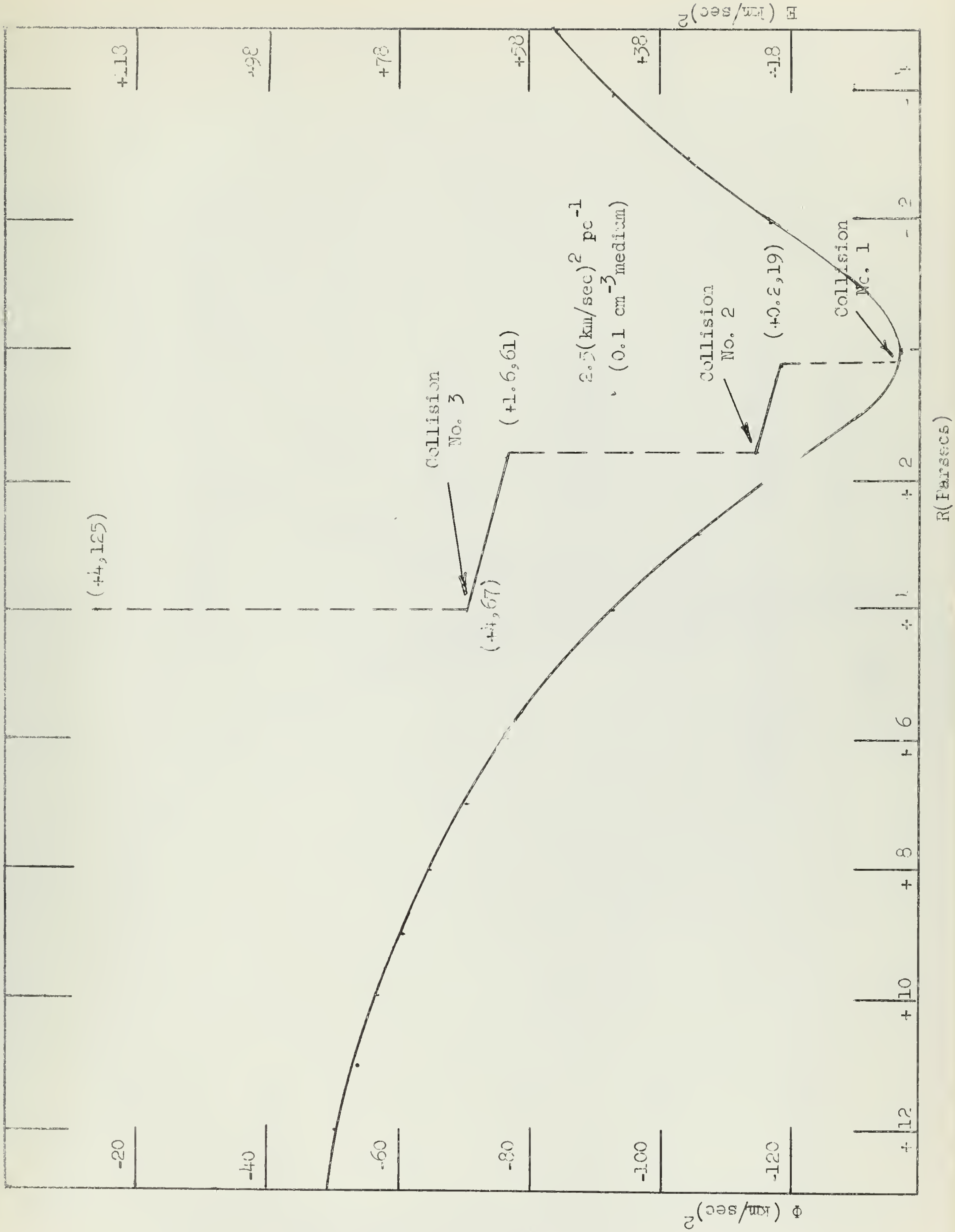

Figure 1 - POTENTIAL WELL FOR M3 SHOWING TYPICAL PASSAGE THROUGH DISK

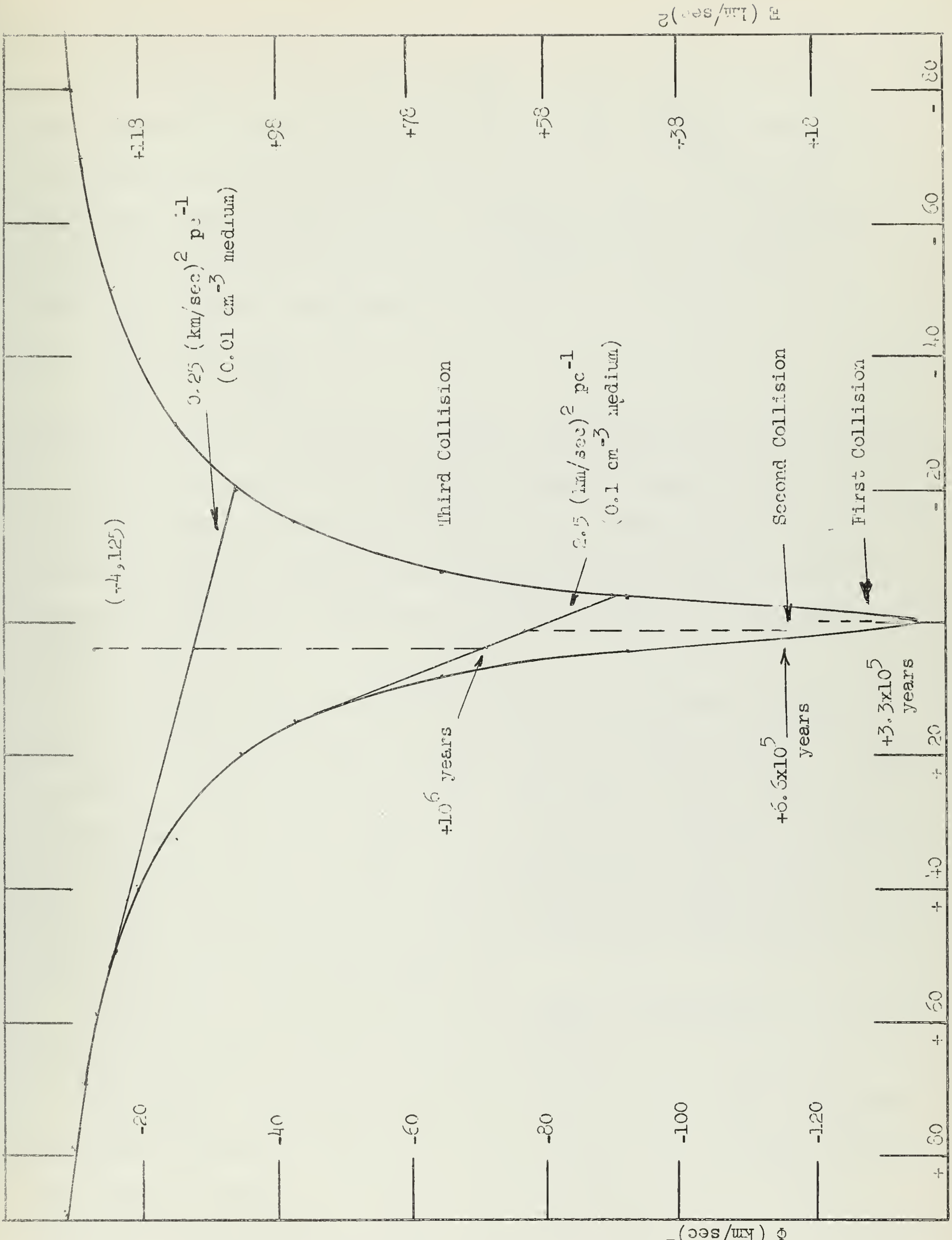

Figure 2 - POTENTIAL WELL FOR M3 to 100 pc

The increment of energy, $\Delta E = 18 \left(\frac{km}{sec}\right)^2$, received by the cluster in the first collision has given the cloud an impetus toward positive R values. We now estimate the distance traversed along this orbit in a time interval, 3×10^5 years, until the next collision. We integrate numerically along the path from $R_1 = 0.2$ pc to an R value that corresponds to a time interval of 3×10^5 years. We use distance steps of 0.2 pc:

$$\int_{R_1}^{R_1 + n(0.2)} \frac{dR}{\mathcal{V}(R)}$$

We then find that the cloud will be at (+1.6, 22) at the time of second collision.

As depicted in figures (1) and (2), we follow the cloud through two more collisions. After the second collision and a time interval of 3×10^5 years, we find that a $\Delta E = 58$ $(km/sec)^2$ for the third collision raises the cloud to a very high energy total $\left(E \simeq 125 \ (km/sec)^2\right)$, which means the cloud probably escapes.

We note from figure (1) that if the cloud does not undergo the third collision, it will find itself in an orbit, with the distance limited to about 7 parsecs. From figure (2), we see that a cloud after the third collision is supposedly left in an orbit extending out to about 60 to 70 parsecs. However, it is easy to see that, even for a very small frictional force which could reasonably be expected from the halo medium (10^{-2} to 10^{-4} cm^{-3}), the cloud would be lost from such a high energy orbit.

While we have worked out only this one example in any detail, it is easy to judge the possible reactions in other situations.

For instance, a subsequent pass by our cluster in the example above, which has had two collisions on the previous pass through the disk 10^8 years before, might find its cloud in an orbit at an energy level of about E = 70. The initial velocity, for this case might vary anywhere from about $\mathcal{V}_i = +12$ km/sec through 0 and back to -12. Minus values for velocity indicate a direction for

the cloud in the same direction as the motion of the cluster orbit. Using formulae (4) and (5), we find that ΔE can have the following values for each value of $\mathcal{V}_i$: for +12 km/sec, $\Delta E = +90$; 0 km/sec, $\Delta E = +18$; -12 km/sec, $\Delta E = -54$. Hence, we see that in these 3 cases: the cloud escapes, has its energy raised about 18 $(\text{km/sec})^2$, and has its energy lowered 54 $(\text{km/sec})^2$ respectively. Statistically, we would expect the cloud to be near an extreme point of its orbit and thus find that the change in energy for an impulse is small.

Two other factors are worth brief mention: (1) we could expect some elliptic orbits to ensue, for the reason that a cluster on a subsequent pass could collide with the disk at an angle to the direction of motion of its rectilinear orbit; and (2) we might expect some damping of the orbit during the period the cluster is away from the disk, because of accumulation of additional medium from evolution of stars and the effects of inelastic collisions.

To estimate the chance of a particular cluster cloud getting through the disk without escaping from the cluster, we use the results from the previous discussion and apply the Poisson frequency function:

$$f(k) = \frac{a^k e^{-a}}{k!}$$

where a is the expectation and f(k) is the probability that x = k. For our problem, we take a equal to 3 (we can expect about 3 collisions per pass). We then evaluate the individual probabilities for k = 0, 1, 2 and 3 collisions per transit. We sum up these individual probabilities, using $\frac{1}{2} \cdot f(k)$ for k = 3, and find that the probability of a cluster cloud surviving one disk passage is about $\frac{1}{2}$. Of course this figure is for the average assumed conditions of the disk and would vary for a different structure in any region.

We should note that the foregoing development for momentum transfer applies equally well for the Idlis cloud. That this is true is readily understood from equations (1) and (4) which show that the forces depend on the product rN which are approximately equal for the two cases.

It is clear that any original gases in the globular cluster are swept out at some time during passages through the disk during 10^{10} years. We previously mentioned one other possible source of material for the cluster, that of sweeping up the gases from the halo. Spitzer gives a value of 5×10^{-4} H atoms cm^{-3} for the halo. Shklovsky and Pikelner (1958) give $10^{-2} cm^{-3}$. If, for example, a Roberts cloud swept a path during its orbit of approximately 30 kpc length, and retained all swept matter, the total mass accumulated would be only about 20 $\odot$, even by using $N = 10^{-2}$. We can conclude that a "snow plow" accumulation is a relatively small source of material.

Energy Transfer

In addition to the effects of momentum transfer, we must also consider the heating effects of a collision between clouds at these high energies. From our discussion in Chapter III of heating and cooling processes in cosmic clouds, we learned that if an HI region becomes ionized, it will have an equilibrium temperature of about 10,000 $^{\circ}$K and the root mean square velocity of the protons will be about 16 km/sec. The Oort-van Herk dynamical model for M3 gives a velocity of escape of 16.6 km/sec at the center ranging down to 11.4 km/sec at 8 pc and so on, as can be surmised from figure 2. We conclude that when hydrogen is raised to a kinetic temperature of 10,000 $^{\circ}$K, it will quickly expand and escape from the cluster.

Bates and Griffing (1953) have calculated the ionization cross section for the reaction $H(1S)+H(1S) = H(1S)+H^{+}+e$. Extrapolating their data, we estimate this cross section as $10^{-20}cm^{2}$ for a velocity of about 300 km/sec. Comparing the ionization cross section ($\sim 10^{-20}cm^{2}$) with the collision cross section ($\sim 10^{-16}cm^{2}$), we see that an individual hydrogen atom has a very low probability ($\sim 10^{-4}$) of ionization per collision. The incoming energetic hydrogen atoms, relative to the frame of reference of the dense ($\sim 10^{3}$ to $10^{5}cm^{-3}$) cloud or globule of the cluster, will be quickly damped to energies below 13.6 ev as a result of both momentum transfer and radiation losses by excited atoms. As we will see later, the ionization will be mostly confined to a narrow interface between two colliding clouds.

We should keep in mind that collisional ionization is not a very efficient process. Alfvén (1960) has proposed that collisional ionization between a plasma and a neutral gas is caused by electrons being accelerated to sufficient energies by irregularities in the charge distribution created in the plasma. In the discussion following this paper, Spitzer remarks that when ionization is produced by energetic electrons, the cross section for excitation is much greater than that for ionization and 4 to 5 times as much energy goes into light than

into ionization. Naturally, for any ionization mechanism, a certain proportion of the input energy will be dissipated as light and infra-red radiation from excited atoms and from grains.

We will analyze in some detail the case of collision between a Roberts cloud ($N \sim 10^3 cm^{-3}$) and a typical cloud in the disk ($n \sim 10\ cm^{-3}$, diameter $[L] \sim 7.5 pc$).

First, we should note that at relative velocities of about 300 km/sec, the interacting particles exceed the velocity of sound and we can expect a compressed shock front between two colliding clouds. The velocity of sound is given by $a = \sqrt{\gamma RT}$, where $\gamma = 5/3$, $R = 8.3 \times 10^7$. For $T = 100\ ^\circ K$, $a = 1.2$ km/sec; for $T = 10{,}000\ ^\circ K$, $a = 17$ km/sec.

We now calculate the total energy per second transferred to the more dense Roberts cloud as it passes through the larger cloud of the disk. We have for energy/unit time:

$$E/\text{unit time} = \tfrac{1}{2}(\pi r^2 x V) x (m_c \cdot m_H) x V^2 = 6.4 \times 10^{36} \text{ ergs/sec}$$

(where $m_c = 10\ cm^{-3}$ for disk cloud, $r = 1$ pc for Roberts cloud, $V = 300$ km/sec, $m_H = 1.67 \times 10^{-24}$ gm). This means that $\frac{6.4 \times 10^{36}}{\pi r^2}$ or 0.23 ergs $cm^{-2} sec^{-1}$ is the amount of energy input per unit time per unit area which must be dissipated at the interface of the Roberts cloud. We note that this value is comparable to the average stellar radiation per unit area in a cluster:

$$\frac{uc}{4} = \frac{2.9 \times 10^{-10} x 3 \times 10^{10}}{4} = 0.22 \text{ ergs } cm^{-2} sec^{-1}$$

We should investigate the possible cooling mechanisms which are able to dissipate this energy and maintain the layer at a reasonable equilibrium temperature.

Spitzer (1949) has given the following formula for computing the rate of energy loss caused by the inelastic collisions between hydrogen atoms and dust grains:

$$L_{Hg} \text{ (ergs/cm}^3\text{sec)} = 9.4 \times 10^{-22}\, n(g) N(H) T^{\frac{1}{2}} (T - Tg)$$

We take the following values for:

$n(g)$ - concentration of dust particles = $10^{-8}\,cm^{-3}$

(where radius of particles $\simeq 10^{-5}$ cm and density $\simeq 1$ gm/cm^3)

$N(H)$ - value for Roberts cloud $\sim 10^3\,cm^{-3}$

(the above two values give a mass ratio of dust to gas of approximately 1%)

T - gas temperature assumed to be 10,000 $^{\circ}$K.

T_g - temperature of the grains is assumed very small (10-50 $^{\circ}$K).

Inserting these values in Spitzer's formula, we find $L_{Hg} = 9.4\times10^{-21}$ ergs cm^{-3}sec^{-1}. We have an energy input by collision of 0.23 ergs cm^{-2}sec^{-1}, which even if distributed throughout a layer of several mean free paths for collision ($\lambda = 1/N\sigma = 1/10^3 . 3\times10^{-16} = 3\times10^{12}$ cm) would be many orders of magnitude greater than the cooling rate.

We assume that the impact layer between the two clouds is completely ionized and at a temperature of 10,000 $^{\circ}$K. Accordingly, we will have a recombination of electrons and protons with the accompanying radiation in the visible and infra-red as the electrons cascade down to the ground state. We use the following formula to obtain an estimate of the cooling rate provided by this mechanism:

$$(6)\quad L_{rec.}(\text{ergs cm}^{-2}\text{sec}^{-1}) = N_p N_e v \sum \sigma_n (h\nu)_n . D$$

where N_p = proton concentration in the impact layer (cm^{-3}).

N_e = electron concentration (cm^{-3}).

v = velocity of electron at 10,000 $^{\circ}$K.

σ_n = capture cross section to nth state for hydrogen.

$(h\nu)_n$ = average energy radiated per capture to state n, and in cascade to ground state.

D = thickness of ionized impact layer (cm).

From page 91 of Allen (1955), we take the values given for the recombination cross section to the nth state for hydrogen and multiply each value by $(h\nu)_n$, and then sum all products to give: $\sum \sigma_n (h\nu)_n = 5.7\times10^{-32}$ ergs cm^{-2} (where each σ_n is of the order of 10^{-22} cm^2).

For the velocity of the electron at 10,000 ^{o}K, we use $v = 6.4\times10^{7}$ cm/sec. We consider that $N_p \simeq N_e$ and set the equation for L_{rec} equal to the energy input. We then solve for N^2D:

$$N^2 D(v)(\Sigma \sigma_n (h\nu)_n = \text{energy input from collision}$$

$$N^2 D\ (6.4\times10^{7})(5.7\times10^{-32}) = 0.23 \text{ ergs cm}^{-2}\text{sec}^{-1}$$

(7) or $N^2 D = 6.3\times10^{22}\text{cm}^{-5}$

Having obtained an expression for N^2D, we now find N from considerations of pressure equilibrium in the collision process. We note that the collision picture can be described in the frame of reference in which the ionized shock layer between the two clouds is stationary so that a disk cloud ($n = 10$ cm^{-3}) moves with a velocity of about v = 300 km/sec. We can readily compute the rate of transfer of momentum per unit area at the front by the expression:

$$n.m_H.v^2 = 10\ (\text{cm}^{-3}).1.67.10^{24}(\text{gm}).(3\times10^{7})^2\text{cm}^2\text{sec}^{-2}$$

$$= 1.5.10^{-8} \text{ gm cm}^2\text{sec}^{-2}$$

Since this value is the force per unit area or pressure exerted on the layer from the direction opposed to the direction of motion, we can equate it to our expression for the pressure inside the layer,

$p = N_p kT$ (where N_p = number of protons per cm^3, k = Boltzmann's constant (1.38×10^{-16}ergs/degree), and T=10,000 ^{o}K). We use N_p as the concentration in this expression since the electrons contribute only about 1/43 of the effective pressure. We find $N_p \simeq 10^{4}\text{cm}^{-3}$.

Using our result from equation (7) that $N^2D = 6.3\times10^{22}\text{cm}^{-5}$, we then solve for the thickness of the shock layer, finding $D = 6.3\times10^{14}$cm. We note that D is larger than the mean free path for collision of H atoms ($\lambda = 3\times10^{12}$cm) in the dense cloud and comparable to the mean free path ($\lambda = 3\times10^{14}$cm) in the disk cloud. We also note that it is extremely thin compared to the diameter of the cluster cloud (2 pc).

It follows that we have a situation in which the two clouds and their interface are in pressure equilibrium. Using the expression for momentum transfer per unit area, $n.m_h.V^2$, we see that: $v^2_{cluster} = \frac{V^2 n}{N} = \frac{(300)^2 .10}{10^3}$ or $v_{cluster} \simeq 30$ km/sec (relative to frame of reference where shock layer is stationary).

We then see that the distance of travel by the shock front relative to the cluster cloud during the time of collision is equal to $\frac{30}{300} \times L$ (where L = 7.5 pc). This distance of travel by the shock front (~0.75 pc) is less than the diameter of the cluster cloud (2.0 pc), as we would expect.

The effect of this impact layer is to convert high energy particles into radiation in the visible and infra-red region at the spectrum. This, in turn, serves to increase the radiation density at these wavelengths inside the cloud of the cluster. In order to determine if the cloud can handle this increase in radiation density at these wavelengths, we analyze the cooling capacity of just one mechanism - that of the dust grains. If we should find that an increase in radiation density would require such a high temperature for the dust particles that they could not remain in the solid state, then our cloud could possibly be raised to a high equilibrium temperature. We pursue this analysis in a way which seeks merely to set a limit as to the amount of cooling the dust can handle with respect to the increased energy input.

From Roberts's data we have that the cloud absorbs about 5 magnitudes in the visible region of the spectrum ($\lambda \sim 5 \times 10^{-5}$ cm). If we assume that the dust has a temperature of about 50 °K (at 50 °K, the cloud will be self-gravitating), then from Wien's law: $\lambda_{max.}(\text{cm}) = \frac{0.289}{T(^{\circ}K)}$

$$\lambda_{max} \text{ (at 50 } ^{\circ}\text{K)} \simeq 5.8 \times 10^{-3} \text{ cm}$$

Since the absorption law for small ($\sim 10^{-5}$cm) dust particles shows that absorption varies inversely as wavelength, we conclude that effective absorption

at 50 $^{\circ}$K is approximately 5 mag$\left(\frac{5\text{x}10^{-5}}{5\text{x}10^{-3}}\right) \simeq 0.05$ magnitude. This means that the cloud has an absorption coefficient, $(k) \cong 5\%$, at the relevant wavelength, λ_{max}.

Therefore, we apply the Stefan-Boltzmann radiation law (with the additional factor k) to our cloud, and thereby determine how much energy per second the dust in the cloud can radiate at 50 $^{\circ}$K:

$L(\text{ergs/sec}) = 4\pi r^2 \sigma T^4 . k$

where $r = 1$ pc, $\sigma = 5.7\text{x}10^{-5}$ erg cm^{-2}deg^{-4}sec^{-1},

$T = 50$ $^{\circ}$K and $k = 0.05$.

Substituting these values, we find that $L \cong 2\text{x}10^{38}$ ergs/sec (cooling rate that can be handled by a Roberts cloud with dust particles at a temperature of 50 $^{\circ}$K). Now if we compare this with the maximum possible energy input from the collision process, $6.4\text{x}10^{36}$ ergs/sec, we conclude that the cloud can absorb this additional energy input without vaporizing the dust.

By comparing with the above, the energy input expected in the collision of a cluster cloud with the intercloud medium of 0.1 cm^{-3} concentration, we conclude that the Roberts cloud will not be heated appreciably except for a thin layer at the collision juncture.

We also find that the Idlis cloud can withstand the thermal effects of cloud impact in the galactic disk.

By a similar analysis as used for the 260 $\odot$ cloud, we find that the proton concentration is also 10^4cm^{-3} in the Idlis cloud, making $D \simeq 6\text{x}10^{14}$cm. Using the expression for transfer of momentum per unit area, we find that the ionized layer travels at 0.3 km/sec through the globule ($v^2_{globule} \simeq \frac{(300)^2 . 10}{10^5}$. Since this is below the speed of sound (a = 1.2 km/sec for the material), we do not have a shock phenomenon in this case.

CHAPTER V

DISCUSSION OF RESULTS AND CONCLUSIONS

From the calculations in Chapter IV, we can conclude that a globular cluster will not necessarily lose its interstellar material in passing through the galactic disk.

We have seen that, for a cloud of the proportions described by Roberts and also for the smaller, denser globule of Idlis, momentum transfer will be sufficient in about half the cases to eject it from the cluster. We also learned that a shock front forms during collision with clouds of the galactic disk and that the recombination of hydrogen, and radiation in the infra-red by dust grains is a sufficient cooling process to dissipate the energy of collision which, therefore, will not cause heating and expansion of the cloud.

From considerations discussed in Chapter III, we concluded that the center of the cluster was the most satisfactory environment for HI regions.

Accordingly, it would seem plausible that diffuse material accumulates in the center of the cluster and is excited into orbits at various distances on passages through the galactic disk. We would not expect to see those clouds in the cluster at great distances from the center, even if they occurred there, nor those near the center on photographs with exposures typical of those used by Roberts.

We have examined at some length possible correlations for those 12 clusters (which show obscuring regions) out of the 32 examined by Roberts. Correlations could be expected for at least two reasons: (1) differences in interstellar environments and evolutionary activity among clusters, and (2) significant, systematic differences in the structure of the galactic disk. We must use caution in attempting a correlation since it is not at all certain that the 20 clusters without reported obscuration contain less diffuse material. Roberts has stated (private communication, 1963) that he was conservative in his estimates of

clouds and that his list of 12 could possibly be increased by 50%.

We have noted that only about half of the globular clusters have been reported as having RR Lyrae variable stars (Hogg, 1959). Adding ω Centauri (Fitzgerald 1955) and M4 (Idlis) to Roberts's list of 12 clusters, we note that RR Lyrae stars have been sighted in all but two. Of the 20 clusters examined by Roberts with negative results for obscuring clouds, 13 are listed as having RR Lyrae stars. Generally speaking, the lack of RR Lyrae stars in a cluster is attributed to the absence of stars in the appropriate evolutionary stage. While the above figures don't indicate a significant difference, in this case, some similar criterion having evolutionary significance might possibly be uncovered by a closer search and classification of variable stars and extremely blue stars in globular clusters.

We learned in Chapter IV that collisions with the dense clouds of the disk is the major factor for loss of clouds from the globular clusters. The force exerted by the intercloud medium is not significant. Accordingly, we might expect that if there exists a large, contiguous area of the galactic plane with a significantly smaller proportion of dense ($10\ cm^{-3}$) structure than in the solar neighborhood, that those clusters passing through this region in their galactic orbits would retain more interstellar material and be more likely to show obscurations.

Rougoor and Oort (1960) have remarked that there appears to be a relatively small amount of dense HI structure in the region 600 pc to 3 kpc distance from the center in the galactic plane. Optical observations of the Sb galaxy NGC 2841 (Sandage, 1961) show an amorphous region devoid of dust or spiral structure in the same region (as scaled to our Galaxy).

We have determined, by the use of a formula from Eggen, Lynden-Bell, and Sandage (1962), that the average orbit of a globular cluster ($e = 0.8$) with apogalacticon, 10 kpc, sweeps out about 205° between successive apogalacticon.

We also have determined that our average 0.8 e orbit will pass within 3 parsecs of the center of the Galaxy during disk transit if the semi-major axis lies within a cone with vertex angle of 50°, centered on the galactic axis. We have attempted a correlation, without success, between the present location of clusters within the cone and the presence of obscuring material. The fact that heliocentric distances of globular clusters have probable errors up to 50% makes any such correlation doubtful.

Perhaps the two most significant recommendations for the future study of globular clusters are: (1) An observational program that will determine better the number of hot, blue stars in globular clusters. For example, the incompleteness factor of about 15 for O5 stars in M3 could be improved by more careful observations. These stars are significant in their effect on the ionization of HI clouds, and the radiation field of these stars might be the most important factor influencing the lifetime of HI clouds, since they have a reasonable chance of surviving passage through the galactic disk. (2) It is desirable to have a more detailed program for observing obscured regions in globular clusters. Of primary interest would be the photographing of clusters at many different exposures to determine better the cloud locations and their absorption.

SELECTED BIBLIOGRAPHY

Alfvén, H. 1960, Rev. of Mod. Physics (Symposium on Magneto-Fluid Dynamics), 32, 710.

Allen, C. W. 1955, Astrophysical Quantities (University of London: The Athlone Press), p. 174.

Bates, D.R. and Griffing, G. 1953, Proc. Phys. Soc. (London), 66A, 961.

Dufay, Jean 1957, Galactic Nebulae and Interstellar Matter (London: Hutchinson's Scientific and Technical Publications), p. 243.

Dunham, Theordore Jr. 1939, Proc. Amer. Phil. Soc., 81, 277.

Eddington, A.S. 1930, The Internal Constitution of Stars (New York: Dover Publications, Inc.), p. 371.

Eggen, O.J., Lynden-Bell, D., and Sandage, A.R. 1962, Ap. J. 136, 748.

Fitzgerald, A.P. 1955, Irish Astron. J. 3, 204.

Hogg, H.S. 1959, Handbuch der Physik, edited by S. Flügge (Berlin, Germany: Springer-Verlag), 53, 185.

Hoerner, S. von 1957, Ap. J. 125, 451.

Idlis, G.M. and Nikol'skii, G.M. 1959, Soviet Astronomy, 3, 652.

Jeans, J.H. 1928, Astronomy and Cosmogony (Cambridge: Cambridge at the University Press) p. 340.

Kahn, F.D. 1955, Gas Dynamics of Cosmic Clouds (Amsterdam: North Holland Publishing Co.), p. 60.

Kahn, F.D. 1957, Journal of Fluid Mechanics, 2, 601.

Kahn, F.D. 1958, Rev. of Mod. Phys. (Third Symposium on Cosmical Gas Dynamics), 30, 1069.

King, I. 1962, A.J., 67, 471.

Kinman, T.D. 1959, M.N.R.A.S., 119, 559.

Matsunami, N., Obi, S., Shimoda, B., Takase, B., and Takebe, H. 1959, Pub. Astr. Soc. Japan, 11, 9.

Mayall, N.U. 1946, Ap. J., 104, 290.

Minkowski, R. and Osterbrock, D. 1959, Ap. J., 129, 583.

Oort, J. H. and van Herk, G. 1959, B.A.N., 14, 299.

Parker, E.N. 1958, Rev. of Mod. Physics (Third Symposium on Cosmical Gas Dynamics) 30, 952.

Pickelner, S.B. and Shklovsky, I.S. 1958, Rev. of Mod. Phys. (Third Symposium on Cosmical Gas Dynamics) 30, 935.

Roberts, M. 1959, Nature, 184, 1555.

Roberts, M. 1960, A.J., 65, 457.

Rougoor, G.W. and Oort, J.H. 1960, Proc. Nat. Acad. of Sciences, 46, 1.

Russell, H.N., Dugan, R.S., and Stewart, J.Q. 1938, Astronomy, Vol. II (Ginn and Company).

Sandage, A.R. 1954, A.J., 59, 162.

Sandage, A.R. 1957, Ap. J., 125, 422.

Sandage, A.R. 1961, The Hubble Atlas of Galaxies (Washington, D. C.: Carnegie Institution of Washington), p. 14.

Savedoff, M.P. 1955, Gas Dynamics of Cosmic Clouds (Amsterdam: North Holland Publishing Company) p. 55.

Schmidt, M. 1956, B.A.N., 13, 15.

Spitzer, L. Jr. 1942, Ap.J., 95, 329.

Spitzer, L. Jr. 1948, Ap. J., 107, 6.

Spitzer, L. Jr. 1949a, Problems of Cosmical Aerodynamics, (Dayton, Ohio: Central Air Documents Office), p. 31.

Spitzer, L. Jr. 1949b, Ap. J., 109, 337.

Spitzer, L. Jr. and Savedoff, M.P. 1950, Ap.J., 111, 593.

Spitzer, Lyman Jr. 1954, Ap.J., 120, 1.

Spitzer, L. Jr. 1958, Rev. of Mod. Physics (Third Symposium on Cosmical Gas Dynamics), 30, 1108.

Stromgren, B. 1939, Ap.J., 89, 526.

Stromgren, B. 1948, Ap. J., 108, 242.